

# On the Shapley value of liability games \*

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An insolvent firm (country, state, individual, etc.) has liabilities towards a group of creditors and the asset value of the firm must be distributed among the creditors and the firm itself. In this paper, we investigate properties of the Shapley distribution rule that is the Shapley value of a cooperative game we associate to a liability situation. In a liability game, the worth of a coalition is defined as follows: given a coalition and its complement, the firm first makes payments to the creditors in its coalition, up to the total value of their liabilities or the asset value of the firm, and then (if anything left) pays the rest to the creditors in the complementary coalition.

We present an explicit framework to compute the Shapley value of the liability game directly from the asset value and the liabilities. Although we show that calculating the Shapley payoffs is NP-hard, henceforth its application to large liability problems could become computationally laborious, based on our explicit framework several of its features are easily obtained. In particular, we show that the Shapley distribution rule allocates the asset value non-negatively among the creditors and the firm (efficiency) in such a way that no creditor gets more than his liability or the asset value (truncated liabilities boundedness). Moreover, at the Shapley rule, the insolvent firm ends up with strictly positive equity if and only if it has multiple positive liabilities. We also prove that creditors with higher claims get higher payments, but they also give higher debt forgiveness to the firm.

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