

Monotonicity properties and the nucleolus in assignment games

Tamás Solymosi

CUB

Abstract

The nucleolus is a well-known single-valued solution to transferable utility cooperative games. It has many desirable properties, but in general it might fail to satisfy certain types of monotonicity. It can happen (even among 4-player convex games) that an increase in the value of a coalition, while keeping all other coalitional values fixed, results in a strict decrease in the nucleolus payoff to a coalition-member. There are classes of games, however, in which it is typically not possible to change only one of the coalitional values, since they are determined from externally given data. In such games the proper question is whether or not the payoffs at a point-valued solution responds in a monotonic way to changes in the underlying data. In the talk we discuss these question on the class of assignment games.

Assignment games are models of two-sided one-to-one matching markets with transferable utility. Since only bilateral cooperations can generate added value, an assignment game is completely determined by the matrix consisting of the profit values of all possible mixed pairs of players. We consider various monotonicity concepts on assignment games (aggregate monotonicity, coalitional monotonicity, marginality, strong monotonicity, weak monotonicity, and pairwise monotonicity) and discuss their "meaningfulness" or "applicability" for assignment games. We show that the nucleolus satisfies all above listed properties but marginality, strong monotonicity, and weak monotonicity in assignment games. We conclude that the most suitable monotonicity notion for assignment games is pairwise monotonicity, where if we increase one entry in the profit matrix but keep all other entries fixed, the payoff cannot decrease for either the row or the column player in the corresponding assignment game. We prove that the nucleolus is pairwise monotone for assignment games.