How to divide fairly a cake among several people?

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An unceasing problem of prevailing society is fair division of goods. Therefore, it is essential that the mathematical models behind these procedures lie on a solid basis. The problem of fair cake cutting is to divide a divisible and heterogenous resource in a given ratio, where each player strives to receive as much of the resource as she can. The difficulty of this task is that the players value the pieces differently, thus in a fortunate case everyone could receive bigger piece than her fair share.

In this research, some new fair division algorithms were created. The motivation to construct the equally dividing Happiness in Unity Algorithm (HUA) was to develop the principles of the otherwise optimal Divide and Conquer Algorithm (DCA) further, so as to eliminate the role of the passive player on one hand and to give every player equal rights to get more than their fair share on the other hand. The division is strong fair (so it guarantees for everybody more than her fair share) if everyone decides to cut the cake in different places at the first round. This property is achieved by expanding the number of cuts with a linear content hence it does not change the optimal $O(n \cdot \log n)$ complexity. The k-person Stock Company Splitting Algorithm (SCSA(k)) (on a cake of total value n) successfully solves the unequal cake division problems for k participants. Similarly, we can compare it with the Divide and Conquer Algorithm. Fair division with unequal shares can be reduced to DCA, so unequal division can be accomplished with $O(n \cdot \log n)$ cuts. The complexity of SCSA(k) is $O(2^k \cdot \log n)$, thus it can be proved that when the cake has total value $n > 2^k$ then SCSA(k) performs better than DCA. Note that the estimate on the number of cuts of SCSA(k) is not sharp.

Our future research objective is to find a sharp estimate for the number of cuts of SCSA(k) so as to achieve the complexity of the Multiplayer Unequaly Divider Algorithm (MUDA) in the Robertson-Webb book that uses $O(k^2 \cdot \log n)$ cuts.