

# Matching with Consecutive Acceptance Intervals

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## Abstract

We consider a one-to-one matching market where each agent is to be matched to an object from the set of acceptable objects for this agent and the acceptable set is consecutive with respect to a fixed “objective” ranking of the objects. The interval profile (i.e., the profile of consecutive acceptance intervals) is assumed to be commonly known. Each agent also has a strict individual preference ranking over the objects in her acceptance interval, which is independent of the common ranking of the objects and assumed to be private information.

First we study maximum matchings, which depend on the the interval profile only, and exploiting the structure of consecutive acceptance intervals we propose a simple algorithm for finding a maximum matching at each interval profile. We generalize this algorithm to find all maximum matchings. Maximum matchings can be treated as endowments for exchanging the objects based on the preferences in a strategyproof manner, using a top trading cycles algorithm.

Trading rules treat the objects asymmetrically and, while they provide a large and flexible set of rules, we also wish to explore the existence of matching rules that depend on the agents’ preferences only and not on the labeling of objects. In order to treat objects symmetrically, our main objective is to find maximum Pareto-optimal matching rules with good incentive properties which are also *neutral*. We first show that there is no neutral and strategyproof rule which is both maximum and Pareto-optimal for an arbitrary consecutive interval profile. We then characterize the set of consecutive interval profiles (the *solvable* interval profiles) for which there exist matching rules that satisfy all the required properties. This characterization is based on an algorithm which finds all the different permutations of agents that allow for a serial dictatorship to yield a maximum matching independently of the strict preferences. These permutations are then used as a basis for the matching rules that we call Block Serial Dictatorships, which yield a maximum Pareto-optimal matching at each preference profile. Block Serial Dictatorships are group-strategyproof and neutral for solvable interval profiles, and they are shown to be the only matching rules that satisfy all the desired criteria. We also generalize the Block Serial Dictatorship rules to arbitrary consecutive interval profiles and show that these rules are maximum, Pareto-optimal and group-strategyproof.

Our results generalize some classic results on Pareto-optimal matchings and serial dictatorships when all objects are acceptable.