Popular matchings and self-duality *

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In this talk we consider a relaxation of stability called popularity in a bipartite graph G where every vertex has a strict ranking of its neighbors in some order of preference. A matching M is popular if M never loses a head-to-head election against any matching where each vertex casts a vote. Thus a popular matching is a weak Condorcet winner in the set of all matchings where vertices are voters. A stable matching is a min-size popular matching and there is a simple and efficient algorithm to compute a max-size popular matching in G. However it is NP-hard to compute a max-weight popular matching when there is a weight function on the edge set.

A max-weight popular "mixed matching", which is a lottery or a probability distribution over matchings, could have a much higher weight than any popular matching and such a mixed matching can be computed in polynomial time. But a drawback of generalizing from pure matchings to mixed matchings is that the optimal solution has become more complex to describe and difficult to implement. We show that there is an optimal mixed matching with a simple structure: it is of the form $\{(M, 0.5), (M', 0.5)\}$ where M and M' are matchings in G. This simple structure is due to the self-duality of the linear program that gives rise to the polytope of popular fractional matchings in G.

*Based on a joint work with Chien-Chung Huang.