

# Dynamic cooperation

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## Abstract

We examine the notion of the core when cooperation takes place in a setting with time and uncertainty. We do so in a two-period general equilibrium setting with incomplete markets. Market incompleteness implies that players cannot make all possible binding commitments regarding their actions at different date-events. We unify various treatments of dynamic core concepts existing in the literature. This results in definitions of the Classical Core, the Segregated Core, the Two-stage Core, the Strong Sequential Core, and the Weak Sequential Core. Except for the Classical Core, all these concepts can be defined by requiring absence of blocking in period 0 and at any date-event in period 1. The concepts only differ with respect to the notion of blocking in period 0. To evaluate these concepts, we study three market structures in detail: strongly complete markets, incomplete markets in finance economies, and incomplete markets in settings with multiple commodities. Even when markets are strongly complete, the Classical Core is argued not to be an appropriate concept. For the general case of incomplete markets, the Weak Sequential Core is the only concept that does not suffer from major defects.

In the second part we introduce the concept of a transferable utility game with uncertainty (TUU-game). In a TUU-game there is uncertainty regarding the payoffs of coalitions. One out of a finite number of states of nature may materialize and conditional on the state, the players are involved in a particular transferable utility game. We consider the case without ex ante commitment possibilities and propose the Weak Sequential Core as a solution concept. We characterize the Weak Sequential Core and show that it is non-empty if all ex post TU-games are convex. We study bankruptcy games with uncertainty and apply the Weak Sequential Core. We find that most of the best-known allocation rules are unstable in this setting, except for the Constrained Equal Awards rule.

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