The Hamiltonian Cycle Problem And Some Challenging Non-Convex Programs

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Abstract

We consider the famous Hamiltonian cycle problem (HCP) embedded in a Markov decision process (MDP). More specifically, we consider the HCP as an optimization problem over the space of state-action frequencies induced by the MDP's stationary policies. In recent years, this approach to the HCP has led to a number of alternative formulations and algorithmic approaches involving researchers from a number of countries including Australia, USA, India, The Netherlands, France, China and Russia.

In this lecture we focus on approaches involving certain classes of perturbed and unperturbed mathematical programs. One of these is a suitably constructed indefinite quadratic programming problem over a polytope. It is known that whenever a given graph possesses Hamiltonian cycles all global minima of this indefinite program are attained at extreme points of the feasible region induced by these cycles. Also, the nonnegative objective function attains the lower bound of zero at these global minima. We present a "Branch & Fix" type algorithm that solves the HCP (and in the process the above global optimisation problem for Hamiltonian graphs). At each branch of the algorithm, only a linear program needs to be solved and the dimensions of the successive linear programs are shrinking rather than expanding. Another optimization problem is that of minimizing the variance of first return times to the home node. The latter, in turn, leads to an interesting problem of minimizing the determinant of a rank-one corrected generator of a Markov chain. All of these formulations lead to some challenging, still unsolved, problems.