

# Monotonicity of scoring methods for preference aggregation

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## Abstract

We review some properties of ranking models used for generalized tournaments with missing values and multiple comparisons. Several scoring procedures for this problem have been proposed, some of them were also characterized by a set of axioms. Nevertheless, the selection of the appropriate method is still challenging. An axiomatic approach is taken with a focus on the independence of irrelevant alternatives, the properties describing monotonicity and the possible ways of manipulation. It is a basic issue both for theoretic research and practical applications, especially in sport and other fields, where the alternatives (players) are able to influence the paired comparison results.

Based on the discussions of various ranking methods, some relevant axioms are defined and their connections are presented. It is revealed that the structure of the comparisons have some implications for monotonicity. We introduce the new concept of manipulation by coalitions. The properties are investigated with respect to three relating scoring methods, the score (Borda / row sum), the generalized row sum (which is itself a parametric family of methods) and the least squares procedures. All of them can be obtained through a system of linear equations, making the proofs simpler. Most results are affirmative, while negative findings can be traced back to inherent contradiction among the axioms. The collection of main properties can contribute to the comparison of ranking methods and to the characterization of them as well as support the planning of schedule in incomplete and imbalanced tournaments.

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