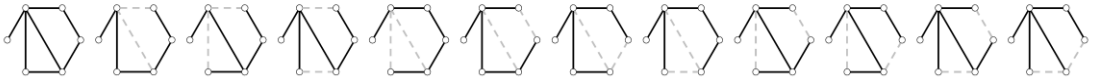

Spanning trees and logarithmic least squares optimality for complete and incomplete pairwise comparison matrices

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Pairwise comparison matrices provide a user-friendly way of cardinal preference modeling. Decision makers compare the importance of criteria, or the performance of alternatives with respect to a given criterion. Numerical answers are arranged into a square matrix, which is elementwise reciprocal of its own transpose. A pairwise comparison matrix can be completely filled in (complete) or incomplete. Incomplete pairwise comparison matrices offer a wider range of applicability, not only in multi-criteria decision making, but in ranking problems as well.

The objective is to determine weights that express the importance of criteria, or the scores of the alternatives with respect to a criterion, by numbers, such that the pairwise ratios of the weights are *as close as possible* to the matrix elements, given by the decision maker. Several distance minimizing methods have been proposed, as well as other methods without the specification of the metric. The spanning tree approach belongs to the second group by definition. However, Lundy, Siraj and Greco proved in a recent EJOR paper (DOI 10.1016/j.ejor.2016.07.042) that the geometric mean of the weight vectors, calculated from all spanning trees of a complete pairwise comparison matrix's graph, is in fact the optimal solution of the logarithmic least squares problem.

We generalize this result for the class of incomplete pairwise comparison matrices.

The detailed proof can be found at https://arxiv.org/abs/1701.04265