The Stable Fixtures Problem with Payments

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Abstract

We introduce multiple partners matching games, which consist of a graph $G = (N, E)$, with an integer vertex capacity function b and an edge weighting w. The set N consists of a number of players that are to form a set $M \subseteq E$ of 2-player coalitions ij with value w_{ij}, such that each player i is in at most bi coalitions. A payoff p is a vector with $p(i,j) + p(j,i) = w_{ij}$ if $ij \in M$ and $p(i,j) = p(j,i) = 0$ if $ij \notin M$. The pair (M, p) is called a solution. A pair of players i, j with $ij \in E \setminus M$ blocks a solution (M, p) if i, j can form, possibly only after withdrawing from one of their existing 2-player coalitions, a new 2-player coalition in which they are mutually better off. A solution is stable if it has no blocking pairs.

We give a polynomial-time algorithm that either finds that no stable solutions exists, or obtains a stable solution. This generalizes a known result of Sotomayor (1992) for multiple partners assignment games, where the underlying graph G is bipartite. We also characterize the set of stable solutions and initiate a study on the core of the corresponding cooperative game, where coalitions of any size may be formed.