Pairwise comparison matrices and optimization

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In multi-attribute decision making pairwise comparison matrices can be used to obtain weights of criteria, or performances of alternatives with respect to a single criterion. Decision makers do not need to give these values directly, they rather compare the importance or performance of criteria or alternatives in pairs. This is done on a ratio scale. The resulting comparisons are arranged in a pairwise comparison matrix (PCM), which is a positive, reciprocally symmetric matrix. The weight vector is calculated from this matrix.

The first topic is Pareto-optimality. The elements of the PCM are approximations of the ratios of the corresponding weights. A weight vector is Pareto-optimal with respect to a given PCM, if it cannot be modified in order to be closer to the corresponding element, for at least one of the ratios, in the PCM, without making the approximation of some other element worse. One of the most popular methods to obtain a weight vector, the eigenvector method, which gives the weight vector as the principal right eigenvector of the PCM, does not always produce a Pareto-optimal result. However, we managed to prove that the eigenvector method always gives a Pareto-optimal result, if the PCM differs from a consistent one in at most two elements.

The second topic is calculation of the principal eigenvector of PCMs. The first method is based on the Newton-method, and can be used to obtain the weight vector of incomplete PCMs, which is a PCM with missing elements. The second method, based on the Collatz-Wielandt formula and the method of cyclic coordinates calculates the principal eigenvector of positive matrices.